Equivalent permeability model to simplify the nonlinear hysteretic B-H curve in time harmonic FEM simulations of induction heating

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This paper addresses different levels of simplification for non-linear magnetic B-H curves intended to be used in induction hardening FEM simulations. The magnetic permeability is used in the time domain as a reference and compared to frequency domain equivalences. The error made by ignoring harmonic content in time harmonic simulations is quantified from an energy point of view. Models such as the co-energy density are shown. We present the idea of fitting an equivalent permeability from simulated results using the most detailed material properties available. Even magnetic hysteresis is taken into account thanks to an in-house FEM code. We calculate the power density distribution from eddy current losses and the temperature distribution for a given time period. The optimal magnetic permeability curve used in the frequency domain is fitted by the power density to mimic the temperature in the steel part.

Index Terms—**Magnetic hysteresis, Eddy currents, Electromagnetic induction, Heat treatment.**

I. INTRODUCTION

THE community of induction heating often uses finite element simulations to predict temperatures throughout the process. Good prediction shortens the development time and cost of heat treatment recipes on complex geometries of steel parts [1]. Using the right material properties is crucial at this stage, but the conversion of experimental data into a useful simulation model is not straightforward, as shown below.

It is customary in this field that a fully transient scheme is used for solving the heat equation, and to couple the transient solver with a less computationally demanding time harmonic electromagnetic solution for calculating the heat generation that feeds the thermal problem. The electromagnetic problem is considered as a steady state throughout the thermal time steps due to large differences in timescales. Further simplifications use a linear magnetic permeability, which cannot mimic precisely the nonlinear electrodynamics occurring in ferromagnetic materials. This simplified model differs substantially from a full nonlinear electromagnetic simulation of Maxwell equations.

In this paper, we try to formalize the comparison between a complete nonlinear and hysteretic B-H curve model solved with nonlinear FEM and some simpler magnetic models in order to determine if a formal rule could be defined to find an equivalent linear magnetic permeability from the experimental B-H curve. This model should be usable reliably in many contexts of induction heating, keeping in mind that the main point of comparison is the power and/or temperature profile in the work piece as a function of time.

II. TOWARDS A BETTER TIME HARMONIC MODEL

A. Classical FEM simulations of eddy current problems

In many formulations of Maxwell's equations coupled with nonlinear constitutive relationships, the amplitude of the magnetic flux density B is related to that of the magnetic field H through the nonlinear and possibly hysteretic magnetic relative permeability $\mu_r(H)$. When performing a transient simulation based on such a model, we get the most accurate results on eddy current problems since the magnetic field is calculated at every time step in the fine details. A time harmonic simulation can obtain this steady state result when a full spectrum of frequencies is considered. Simplification, through truncation and indirect coupling, leads to the simplest form: solving at the excitation frequency. In this case, μ_r is locally constant by definition, and any distortion in the excitation signal or magnetic response of the material will be poorly approximated if operating near the saturation point of steel. The use of simple time harmonic simulations provides substantial computational speed though, especially because in induction heating, many other nonlinear physics are present, namely nonlinear thermal and metallurgical processes [2].

B. Approach to define an effective permeability

In this research, we utilize an effective magnetic permeability to express non-linear behavior, hysteresis and bi-frequency applications. The basic idea is to simulate the diffusion of the magnetic field as precisely as possible and then calculate the resulting power density profile. Only then can we fit an effective permeability that will approximate the actual results in terms of temperature.

The time domain simulation used for establishing the equivalence is a 1-D semi-infinite slab problem. Time

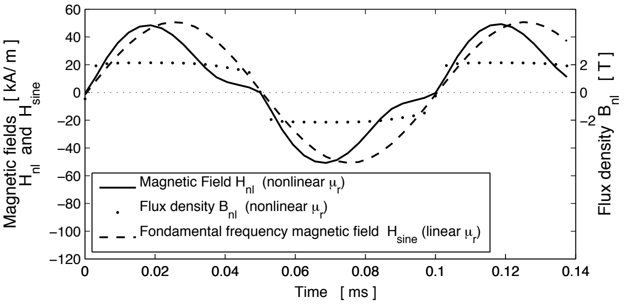

Fig. 1. Typical magnetic saturation observed at a depth of 0.225 mm in a ferromagnetic steel slab when submitted to a sinusoidal applied field. The two first curves were obtained with a full nonlinear time transient simulation. The third curve comes from a linear time harmonic simulation.

transient simulations are realized with an in-house FEM code solving an H-formulation of the Maxwell equations with non-linear permeability and hysteresis through a Preisach model. This is not standard in the induction heating area. Using a 1-D model allows us to avoid any problem related to geometric effects and demagnetization factors, which we shall treat in further work. We assume validity in 3D by defining the permeability as a function of H in an isotropic material as is the case with our steel parts.

From the reference results obtained above with a full nonlinear model, we performed a series of time harmonic simulations with various effective magnetic permeability models until we obtained similar results in the frequency domain and in the time domain in terms of power density profile in the slab. As a first step, the permeability is uniform in the whole ferromagnetic domain, so the time harmonic problem is fully linear. Obviously, the fitting error achieved by this method is considerable and works for a limited range of surface amplitudes, but it provides a good starting point for defining an effective permeability (see Fig. 3 for example).

Once the power density distribution is obtained, various models can be used in conjunction with an effective permeability curve to make sure we obtain corresponding power density profiles, and thus the same amount of energy transferred locally to the work piece being heated. A large family of models defines a magnetic permeability that is constant over a given time step, which in this case refers to the thermal problem, but varies in space (non-uniform permeability). In other words the permeability is locally linear, but it keeps a dependence on the local amplitude of the magnetic field. In this work, we focus our efforts on these types of models due to a good balance between faster computation speeds and precision while maintaining compatibility with a full domain FEM simulation, contrary to a surface impedance model [3].

C. Co-energy based permeability model

The co-energy model is an example of these models [4]. It is interesting since the choice of equivalent linear permeability is based on the power transmitted to the material similarly to our objective. The effective permeability is fixed throughout the time step of a harmonic simulation, but its value can obviously not be taken be directly from the DC hysteresis curve.

The co-energy density technique integrates the nonlinear B-H curve for a quarter cycle giving w_{1i} (see Fig. 2), where i identifies the subdomain of interest, typically one element of a finite element mesh. Then we average this value with the area w_{2i} created by the linear permeability that links the field amplitude H_{mi} with the corresponding flux density B_{mi}, i.e.

$$\frac{\mu_{eq}H_{mi}^2}{2} = \frac{w_{1i}+w_{2i}}{2}. \tag{1}$$

The equivalent curve can be easily calculated from the experimental *B-H* curve and once formatted, it can be used in a time harmonic simulation, which is now nonlinear since the permeability is non-uniform, but it is still a static problem. Figure 4 shows the importance of this formatting.

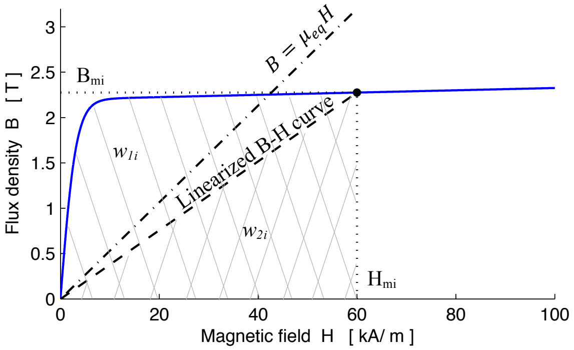

Fig. 3. Typical anhysteretic B-H curve of steel used in transient simulations with the illustration of the co-energy density equivalent permeability.

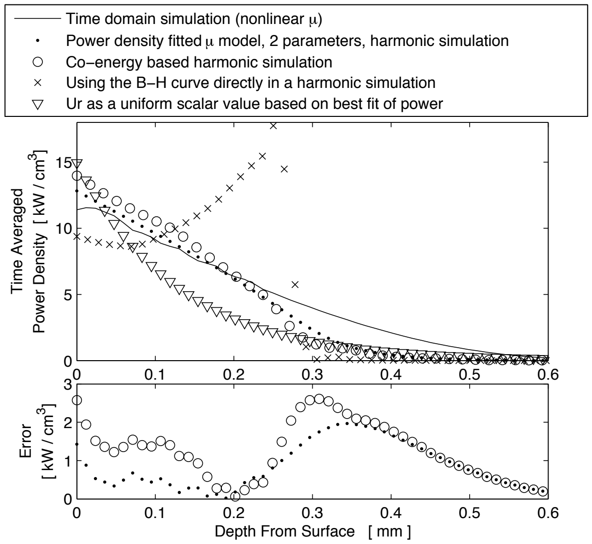

Fig. 4. The co-energy model and the simple power density fitted model used in a time harmonic simulation significantly reduce the error on the power density profile obtained with a full nonlinear transient simulation as compared to using a well choosen uniform scalar value or the linearized B-H curve.

III. RESULTS

In the full version of the paper, we discuss the efficacy of various objective functions for fitting the power density profile and extract the effective permeability curve from experimental B-H curves that include both hysteresis and saturation. A non-uniform permeability is used in all cases of time harmonic simulations. The resulting curve shall be ready to use in multiple types of eddy loss simulations. We will also explore the applicability of this effective permeability model for multi-frequency induction hardening.

IV. REFERENCES

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